

SECURITIES CLASS ACTIONS

Trading Models to Estimate Individual Investor Trading Activity and Aggregate Damages

By Ioannis Gkatzimas, Yingzhen Li, and Torben Voetmann of The Brattle Group, Inc.

In securities class action lawsuits, the amount of aggregate damages defendants may face is of interest to all parties involved in litigation. Both plaintiffs and defendants have traditionally relied on trading models as a tool to assess defendants' liability.

As the overwhelming majority of securities class action cases lack access to individual investor trading data, parties must estimate aggregate damages.¹ Litigants have been using trading models to estimate damages in securities litigation for more than two decades. A trading model incorporates assumptions about investors' trading patterns and simulates the trading activity of market participants using a variety of simplifying assumptions. The aggregate damages estimate based on a trading model can oftentimes drive the course of the litigation.² This article reviews the basic intuition behind trading models.³

BACKGROUND

Consider a hypothetical securities class action lawsuit against Company XYZ, involving a single alleged misstatement or omission and a single alleged disclosure, as illustrated in Figure 1. The pre-Class Period is labeled as Period A. Period B represents the alleged Class Period during which the stock price may have been inflated due to misstatements or omissions. At the end of Period B, a single corrective disclosure allegedly removes any price inflation and causes the stock price to decline.⁴ The 90-day post-Class Period is labeled as Period C. The basic assumption is that the only shares eligible for damages are those purchased during Period B and retained at least until the beginning of Period C.⁵

¹ In cases of less liquid securities with limited numbers of transactions, it is plausible that a complete trading record of each investor may be compiled based on data produced during the discovery process. However, even when the complete trading record is available, additional assumptions may be needed for investors that transacted multiple times during the Class Period.

² The aggregate damages estimate based on a trading model is often labeled "plaintiff-style aggregate damages estimate" because it is an approximation of the true class-wide damages. Trading models make simple assumptions about the actual purchase date of shares sold during the Class Period or retained at the end of the Class Period. The actual purchase dates can only be ascertained from trading records or trading information for each individual plaintiff during the claims process, i.e., each investor has to demonstrate proof of claim.

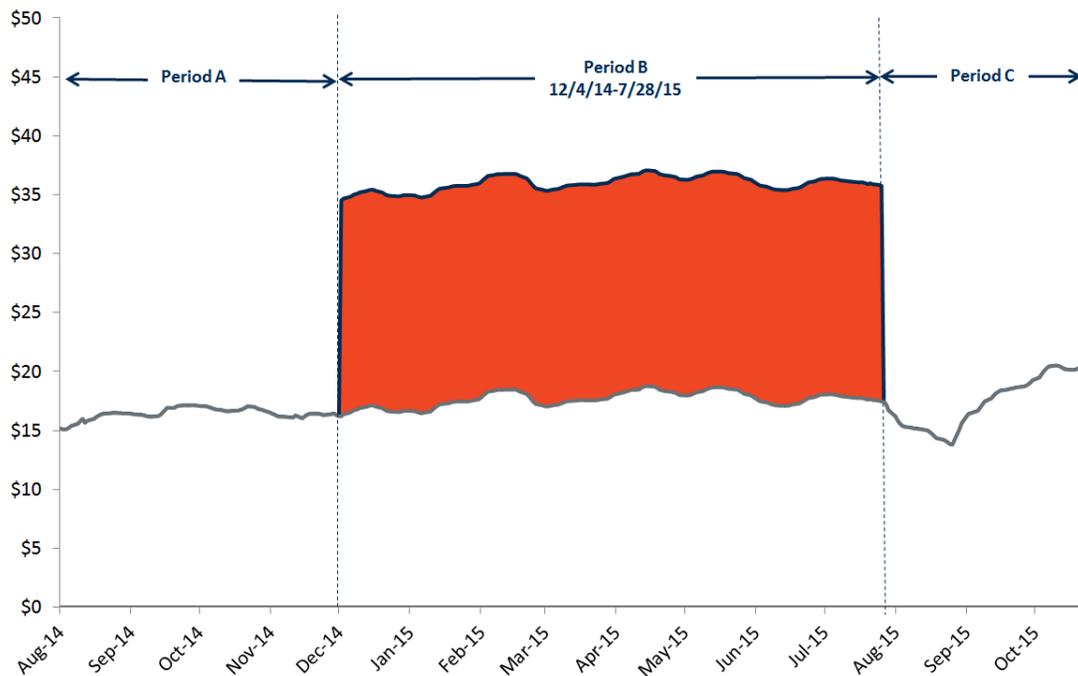
³ The trading models discussed in this article are also applicable for class actions involving fixed income or other types of securities, assuming data is available.

⁴ The price decline attributable to alleged misrepresentations or omissions is commonly referred to as "inflation," which is used to determine the "true" value of the security absent the alleged fraud on each day during the alleged Class Period.

⁵ In the case where there are multiple disclosures made during the alleged Class Period, certain shares that are purchased and sold during the alleged Class Period may also be eligible for damages. Damages from those shares are commonly referred to as "In-and-Out Damages." The In-and-Out Damages are based on shares that are purchased during the Class Period and sold after a corrective

Trading models estimate the holding period of shares by using daily reported trading volume combined with assumptions about the trading patterns of individual investors. For a publicly traded company, stock exchanges report a daily trading volume that aggregates trading of shares between market participants. Trading models allocate daily volume to identified pairs of purchase and sale dates within an alleged Class Period,⁶ allowing for an estimate of investor trading activity. As a result, trading models allocate daily trading volume to investors grouped by their original purchase date.

Figure 1 Hypothetical Example One
Stock Price Movement of Company XYZ



A trading model requires data on (i) a daily estimate of the total number of shares outstanding and available to trade (also known as the float)⁷ and (ii) the daily trading volume associated with a security of interest or, equivalently, the investors eligible for damages.

disclosure but before the final corrective disclosure. Trading models track shares eligible for In-and-Out Damages and shares eligible for Retention Damages (shares that are held until after the final corrective disclosure).

⁶ Under the Private Securities Litigation Reform Act of 1995 (PSLRA), the damages analysis is extended to 90 days after the end of the final corrective disclosure to allow for a bounce-back or recovery in the stock price.

⁷ The float represents the number of shares of a publicly traded company that are eligible to trade (issued and outstanding). The float is different from the number of shares outstanding because it controls for shares held by insiders, and shares that were bought during the Class Period and retained until the final corrective disclosure from shares that were disposed before the fraud was disclosed.

Table 1 below shows data for Hypothetical Example Two with a five-day Class Period from 9/1/2015 to 9/5/2015, where the daily data on “Float” and “Daily Volume,” represent the input data for a trading model.

**Table 1 Hypothetical Example Two
Input Data for Trading Models for A Five-Day Class Period**

| Date | Float | Daily Volume |
|-------------|--------------|---------------------|
| Pre-Class | 100,000 | |
| 9/1/15 | 100,000 | 2,800 |
| 9/2/15 | 100,000 | 1,800 |
| 9/3/15 | 100,000 | 2,050 |
| 9/4/15 | 100,000 | 2,400 |
| 9/5/15 | 100,000 | 2,075 |

Estimating a trading model also requires making an assumption about the likelihood of different investors’ trading on any given day of the Class Period. This assumption is known as the propensity to trade (or to re-trade) and it forms the basis of the method that matches sales to purchases. The propensity to trade assumption is a key distinction among trading models.

ONE-TRADER MODEL

The One-Trader Model, also known as the Proportional Trader Model, assumes that each share is equally likely to be traded on any given day, i.e., each day’s aggregate trading volume is assumed to be comprised from the holdings of all shareholders on a pro-rata basis. The underlying assumption is that all investors exhibit an identical propensity to trade, as if there were a single type of trader representative of all investors in the security.

Trading models simulate market participants’ trading activity sequentially starting from the first day of the Class Period. Table 2 highlights the underlying calculations in a One-Trader Model based on the hypothetical data in Table 1.

In Step [1], all purchases that occurred on 9/1/2015, the first day in the Class Period, become part of the class. These shares must have come from the pool of shares available to be traded (i.e., the pre-Class float). Step [2] calculates the number of shares that have not yet been traded from the pre-Class float (i.e., shares held by investors since before the beginning of the Class Period and have not yet been traded during the Class Period),⁸ i.e., these shares have not entered the class as of the end of 9/1/2015.

⁸ A typical assumption of a trading model is that each share can trade at most once on any given day. To the extent that, in reality, there are day traders who trade in-and-out during a day, the trading volume can be adjusted downward to reflect the idea that those shares are typically not considered to have been damaged.

Table 2 One-Trader Model

| Sell Date | Daily Volume | Pre-Class | Buy Date | | | | | Float |
|-----------|--------------|-------------|-----------|-----------|----------|----------|----------|---------|
| | | | 9/1/2015 | 9/2/2015 | 9/3/2015 | 9/4/2015 | 9/5/2015 | |
| Pre-Class | | 100,000 | | | | | | 100,000 |
| 9/1/2015 | 2,800 | (2,800) | | | | | | |
| | | 97,200 [2] | 2,800 [1] | | | | | 100,000 |
| 9/2/2015 | 1,800 | (1,750) [6] | (50) [4] | | | | | |
| | | 95,450 [7] | 2,750 [5] | 1,800 [3] | | | | 100,000 |
| 9/3/2015 | 2,050 | (1,957) | (56) | (37) | | | | |
| | | 93,494 | 2,693 | 1,763 | 2,050 | | | 100,000 |
| 9/4/2015 | 2,400 | (2,244) | (65) | (42) | (49) | | | |
| | | 91,250 | 2,629 | 1,721 | 2,001 | 2,400 | | 100,000 |
| 9/5/2015 | 2,075 | (1,893) | (55) | (36) | (42) | (50) | | |
| | | 89,356 | 2,574 | 1,685 | 1,959 | 2,350 | 2,075 | 100,000 |

Note:

Step [1]: All of the purchases on 9/1, the first day of the Class Period, enter the class from the pool of potentially damaged shares.

Step [2]: Shares that have not entered the class as of the end of 9/1: 97,200 = 100,000 - 2,800.

Step [3]: All of the purchases on 9/2 enter the class.

Step [4]: Number of shares purchased on 9/1 and sold on 9/2: 50 = 1,800 x (2,800/100,000).

Step [5]: Number of shares purchased on 9/1 but have not been sold as of 9/2: 2,750 = 2,800 - 50.

Step [6]: Number of shares from the float and sold on 9/2: 1,750 = 1,800 x (97,200/100,000).

Step [7]: Number of shares in the float but have not been sold as of 9/2: 95,450 = 97,200 - 1,750.

The One-Trader Model applies the same steps iteratively through each of the trading days in the alleged Class Period by allocating trading volume to shares that are being traded for the first time in the Class Period and shares that are being re-traded within the Class Period. Of the company's float of 100,000 shares, a total of 1,800 shares enter the class on 9/2/15 and become eligible for damages as shown in Step [3]. These 1,800 shares could have come from two possible sources:

- (i) The 2,800 shares that were last traded on 9/1/2015, or
- (ii) The 97,200 shares that have not yet been traded during the Class Period (retained from the pre-Class float) as of the end of 9/1/2015

The fundamental assumption of the One-Trader Model is that these 1,800 shares are equally likely to come from either the shares already traded during the class period or the shares not yet traded on a pro-rata basis, i.e., with equal probability weighted by the number of shares (float). In this example the weights are:

- (i) 2,800/100,000 (2.8%) (shares purchased on 9/1/2015 divided by pre-Class float), and
- (ii) 97,200/100,000 (97.2%) (shares not yet traded in Class Period as of 9/1/2015 divided by pre-Class float)

Step [4] posits that 50 of the shares traded on 9/2/2015 (2.8% of the 1,800 shares) were sold by the traders who had purchased the 2,800 shares on the previous day, on 9/1/2015. As a result, Step [5] deducts these 50 shares from the estimated number of shares purchased on 9/1/2015 and retained through 9/2/2015, updating that number to 2,750. Step [6] likewise removes these 50 re-traded shares from the total of 1,800 shares traded on 9/2/2015, so that 1,750 shares (97.2% of the 1,800 shares traded) are estimated to have had their first Class Period trade on 9/2/2015. Step [7] deducts 1,750 from the running total of shares not yet traded during the Class Period as of the end of 9/2/2015, for an estimated 95,450 remaining untraded shares.

As demonstrated in Hypothetical Example Two, the One-Trader Model allocates trading volume and matches purchases with sales transactions. For example, as shown in Table 2, 2,800 shares are purchased on 9/1/2015, 50 shares of which are sold on 9/2/2015, resulting in 2,750 shares retained as of the end of 9/2/2015. Out of those 2,750 shares, an additional 56 shares are sold on 9/3/2015, resulting in 2,693 shares retained as of the end of 9/3/2015. The calculations discussed above are performed for each subsequent trading day through 9/5/2015 so as to encompass all shares that were purchased during the Class Period and to estimate the shares that were retained after 9/5/2015.⁹

By matching purchases and sales transactions, it is possible to calculate aggregate damages based on the closing price of the security at the end of each trading day and the measure of alleged inflation.¹⁰ In this example, during the 5-day Class Period, 11,125 shares have been purchased,¹¹ 10,644 shares of which are held onto the end of 9/5/2015, the last day of the Class Period.¹² If we assume that each share suffers an inflation of \$5 during the Class Period, the aggregate damages would be $\$5 \times 10,644 = \$53,220$.¹³

This simple example demonstrates the method to determine the combinations of purchases and sales transactions. Once a trading model determines these buy-sale combinations, it is relatively easy for an expert to apply the alleged inflation and calculate aggregate damages in securities class actions.

TWO-TRADER MODEL

The Two-Trader Model allocates ownership of shares and trading behavior among two types of investors. The rationale behind the Two-Trader Model is the idea that there are both active and passive traders in the market. Active traders account for a larger portion of the aggregate trading activity but hold a relatively smaller number of the float on any given date. Passive traders account for a smaller portion of the aggregate trading activity but hold a relatively larger number of outstanding shares or float. A common assumption is to allocate 80% of the trading volume and 20% of the float to active traders, and allocate 20% of the trading volume and 80% of the float to passive traders. Depending on the specifics of the case, different allocation parameters can be used.¹⁴

⁹ The One-Trader Model continues to estimate subsequent sales of shares retained as of 9/5/2015 through the end of the PSLRA 90-day period to estimate the damages of those shares. The Private Securities Litigation Reform Act of 1995, 15 U.S.C. §78u-4(e), states that “the award of damages to the plaintiff shall not exceed the difference between the purchase or sale price paid or received, as appropriate, by the plaintiff for the subject security and the mean trading price of that security during the 90-day period beginning on the date on which the information correcting the misstatement or omission that is the basis for the action is disseminated to the market” and “the ‘mean trading price’ of a security shall be an average of the daily trading price of that security, determined as of the close of the market each day during the 90-day period.”

¹⁰ See *e.g.*, *Robbins v. Koger Properties Inc.*, 116 F.3d 1441, 1449 n.5 (11th Cir. 1997) (“The proper measure of damages utilizes the out-of-pocket rule: the plaintiff can recover ‘the difference between the price paid and the ‘real’ value of the security, i.e., the fair market value absent the misrepresentations, at the time of the initial purchase by the defrauded buyer.”).

¹¹ This is the sum of shares purchased on each day during the Class Period: $2,800 + 1,800 + 2,050 + 2,400 + 2,075 = 11,125$.

¹² This is the total number of shares retained as of the end of the Class Period: $2,574 + 1,685 + 1,959 + 2,350 + 2,075 = 10,644$.

¹³ In this simple stylistic example it is assumed that there is no change in the price of the security during the 90-day period after 9/5/15.

¹⁴ One consideration regarding trading models is whether assumptions about trading propensity and types of investors are a good approximation to actual trading behavior. Users of trading models should adjust aggregate reported trading volume to ensure it does not double-count transactions by market makers and specialists. Recent developments in market microstructure, including increased activity by day-traders (high-frequency traders) who intermediate without establishing positions may possibly require further adjustments to the input assumptions of trading models.

Once the trading volume and the float have been allocated to active and passive traders, the method behind the One-Trader Model is applied separately for each type of trader. Table 3 highlights the underlying calculations for active traders based on the hypothetical data in Table 1.

Table 3 Two-Trader Model: Active Traders

| Sell Date | Daily Volume | Pre-Class | Buy Date | | | | | Float |
|-----------|--------------|-------------|-----------|-----------|----------|----------|----------|--------|
| | | | 9/1/2015 | 9/2/2015 | 9/3/2015 | 9/4/2015 | 9/5/2015 | |
| Pre-Class | | 20,000 | | | | | | 20,000 |
| 9/1/2015 | 2,240 | (2,240) | | | | | | |
| | | 17,760 [2] | 2,240 [1] | | | | | 20,000 |
| 9/2/2015 | 1,440 | (1,279) [6] | (161) [4] | | | | | |
| | | 16,481 [7] | 2,079 [5] | 1,440 [3] | | | | 20,000 |
| 9/3/2015 | 1,640 | (1,351) | (170) | (118) | | | | |
| | | 15,130 | 1,908 | 1,322 | 1,640 | | | 20,000 |
| 9/4/2015 | 1,920 | (1,452) | (183) | (127) | (157) | | | |
| | | 13,677 | 1,725 | 1,195 | 1,483 | 1,920 | | 20,000 |
| 9/5/2015 | 1,660 | (1,135) | (143) | (99) | (123) | (159) | | |
| | | 12,542 | 1,582 | 1,096 | 1,360 | 1,761 | 1,660 | 20,000 |

Note:

Step [1]: All of the purchases on 9/1, the first day of the Class Period, enter the class from the pool of potentially damaged shares.

Step [2]: Shares that have not entered the class as of the end of 9/1: 17,760 = 20,000 - 2,240.

Step [3]: All of the purchases on 9/2 enter the class.

Step [4]: Number of shares purchased on 9/1 and sold on 9/2: 161 = 1,440 x (2,240/20,000).

Step [5]: Number of shares purchased on 9/1 but have not been sold as of 9/2: 2,079 = 2,240 - 161.

Step [6]: Number of shares from the float and sold on 9/2: 1,279 = 1,440 x (17,760/20,000).

Step [7]: Number of shares in the float but have not been sold as of 9/2: 16,481 = 17,760 - 1,279.

As shown in Table 3, active traders account for 20% of the float and 80% of the trading volume shown in Table 2. From these active traders, during the 5-day Class Period, 8,900 shares have been purchased,¹⁵ 7,458 shares of which are held onto the end of 9/5/2015, the last day of the Class Period.¹⁶

Table 4 highlights the underlying calculations for passive traders based on the hypothetical data in Table 1.

¹⁵ This is the sum of shares purchased on each day during the Class Period: 2,240 + 1,440 + 1,640 + 1,920 + 1,660 = 8,900.

¹⁶ This is the total number of shares retained as of the end of the Class Period: 1,582 + 1,096 + 1,360 + 1,761 + 1,660 = 7,458.

Table 4 Two-Trader Model: Passive Traders

| Sell Date | Daily Volume | Pre-Class | Buy Date | | | | | Float |
|-----------|--------------|------------|----------|----------|----------|----------|----------|--------|
| | | | 9/1/2015 | 9/2/2015 | 9/3/2015 | 9/4/2015 | 9/5/2015 | |
| Pre-Class | | 80,000 | | | | | | 80,000 |
| 9/1/2015 | 560 | (560) | | | | | | |
| | | 79,440 [2] | 560 [1] | | | | | 80,000 |
| 9/2/2015 | 360 | (357) [6] | (3) [4] | | | | | |
| | | 79,083 [7] | 557 [5] | 360 [3] | | | | 80,000 |
| 9/3/2015 | 410 | (405) | (3) | (2) | | | | |
| | | 78,677 | 555 | 358 | 410 | | | 80,000 |
| 9/4/2015 | 480 | (472) | (3) | (2) | (2) | | | |
| | | 78,205 | 551 | 356 | 408 | 480 | | 80,000 |
| 9/5/2015 | 415 | (406) | (3) | (2) | (2) | (2) | | |
| | | 77,799 | 548 | 354 | 405 | 478 | 415 | 80,000 |

Note:

Step [1]: All of the purchases on 9/1, the first day of the Class Period, enter the class from the pool of potentially damaged shares.

Step [2]: Shares that have not entered the class as of the end of 9/1: 79,440 = 80,000 - 560.

Step [3]: All of the purchases on 9/2 enter the class.

Step [4]: Number of shares purchased on 9/1 and sold on 9/2: 3 = 360 x (560/80,000).

Step [5]: Number of shares purchased on 9/1 but have not been sold as of 9/2: 557 = 560 - 3.

Step [6]: Number of shares from the float and sold on 9/2: 357 = 360 x (79,440/80,000).

Step [7]: Number of shares in the float but have not been sold as of 9/2: 79,083 = 79,440 - 357.

As shown in Table 4, passive traders account for 80% of the float and 20% of the trading volume shown in Table 2. From these passive traders, during the 5-day Class Period, 2,225 shares have been purchased,¹⁷ 2,201 shares of which are held onto the end of 9/5/2015, the last day of the Class Period.¹⁸

The total number of shares that are held onto the end of 9/5/2015 under the Two-Trader Model is therefore the sum of those from active traders and passive traders, which is 9,659.¹⁹ If we again assume that each share suffers an inflation of \$5, the aggregate damages would be \$5 x 9,659 = \$48,295. This damage estimate is lower than the damage estimate of \$53,220 discussed in the One-Trader Model above. This is the typical case due to the disproportionality between trading volume and float for both types of trader.

As also illustrated in the example above, since the Two-Trader Model involves applying the One-Trader Model separately for each type of trader, it can be extended to account for more than two types of trader (known as the Multiple-Trader Model).

SUMMARY

Trading models are an integral part of estimating aggregate damages in securities class actions, providing a basis for the litigants to discuss settlement and an estimate that the Court can use to determine whether a settlement proposal is reasonable. Therefore, trading models, when applied appropriately, can oftentimes drive the course of the litigation.

¹⁷ This is the sum of shares purchased on each day during the Class Period: 560 + 360 + 410 + 480 + 415 = 2,225.

¹⁸ This is the total number of shares retained as of the end of the Class Period: 548 + 354 + 405 + 478 + 415 = 2,201. (Numbers are rounded.)

¹⁹ This is the total number of shares retained as of the end of the Class Period from active traders and passive traders: 7,458 + 2,201 = 9,659.

AUTHORS



IOANNIS GKATZIMAS

Principal | San Francisco

+1.415.217.1000

Ioannis.Gkatzimas@brattle.com

Mr. Gkatzimas has significant experience in a wide range of finance-related issues including securities class actions, mergers and acquisitions, initial and secondary offerings, structured finance transactions, alternative investments, and on matters related to portfolio performance and attribution. His primary focus is on the valuation of complex securities and portfolios for both commercial litigation and regulatory matters.



YINGZHEN LI

Associate | San Francisco

+1.415.217.1000

Yingzhen.Li@brattle.com

Mr. Yingzhen Li has expertise in financial issues related to securities, capital markets, corporate finance, and financial econometrics. He has conducted economic and financial analysis in a variety of scenarios involving foreign exchange transactions, financial derivatives, credit rating, structured finance, fixed income, securities fraud, market efficiency, debt financing, and financial institutions.



TORBEN VOETMANN

Principal | San Francisco

+1.415.217.1000

Torben.Voetmann@brattle.com

Dr. Voetmann focuses on cases that involve complex economic and financial issues. He has worked with clients and experts on securities cases and valuation disputes related to capital markets and financial institutions. He has analyzed market efficiency, class certification, valuation, mergers and acquisitions, internal investigations, and damages across a variety of cases involving debt, equity, and derivative securities.

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